Introduction. This document contains three guides for preparing short episodes of inquiry-based learning (IBL). The guides given here are about curricular choices and class preparation rather than instructional logistics and mechanics, but Guide 0 lists documents that offer advice on the details of classroom teaching. Inquiry-based learning is executed in lots of different ways and IBL instructors draw from different traditions. Guide 1 draws on the University of Texas-Austin style of IBL, which is often the style assumed if no modifiers are given to the acronym "IBL." Guides 2 and 3 refer to MathILy-style instructor-led class lessons, which we now describe:

> The instructor begins by telling or framing a story that leads to a posed question. The form of the question can be as broad as "So what do you think?" or as narrow as "What should be the definition of 'face'?" or "Can you predict when there will be exactly one owlrabbit in the hatchery?" This is usually followed by a pause of a few seconds to visually read the class status and decide whether to take/solicit student conjectures and discussion or to break students into groups for further work. When student conjectures are made (implicitly or explicitly), some will be used for immediate investigation and others saved for later use. Rarely do MathILy instructors use worksheets as part of a class lesson.
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> As class progresses, instructors take two characteristic actions. One is interrogating student understanding, which means that a student who claims a statement is true or who says ze believes another student's claim is required to justify this both deeply and fully. The other characteristic action is a class check-in. Students grasp different concepts at different rates, and it's easy for quieter or temporarily confused students to get left behind. In a check-in, each student in turn is required to briefly reflect the current mathematical state or to raise a question.

Guide 0: An annotated resource list

Bagnato, Robert A., "Teaching College Mathematics by Question-and-Answer" Educational Studies in Mathematics 5(1)1973:185–192. DOI: 10.1007/BF00684697

Describes a general approach to classroom teaching and how to implement it.

belcastro, sarah-marie. Preface for Instructors and Other Teachers, Section 2.2, *Discrete Mathematics with Ducks*, AK Peters (CRC Press), 2012.

Explains in detail the mechanics of conducting group work in class.

Green, Elizabeth, *Building A Better Teacher*. W.W. Norton, 2014.

A treatise on elementary mathematics education in the U.S. that is somehow applicable in various ways to the teaching of college mathematics.

Braun, Bremser, Duval, Lockwood, White. Active Learning in Mathematics, Part III: Teaching Techniques and Environments, http://blogs.ams.org/matheducation/2015/10/01/active-learning-in-mathematics-part-iii-teaching-techniques-and-environments/#more-985

Gives an overview of various active learning techniques and explains how to incorporate them into different kinds of classrooms.

Guide 1: How to make a concept into a worksheet
(a.k.a. how to write a Texas-style problem list from a curricular goal)

1. Decide on a curricular goal. For example, "students will prove Lagrange's Theorem."

2. List the conceptual ingredients needed for this curricular goal. For example, students need to understand subgroups, cosets, and partitions in order to prove Lagrange's Theorem.

2.5. Do the students have a handle on the necessary conceptual ingredients? If not, make separate curricular goals for the acquisition of each missing concept. This is an excellent way to consider whether, and how, the current curricular goal builds on students' previous knowledge.

3. Write out the mathematical narrative for yourself. Within this narrative, identify the conceptual chunks and any sticky details. Be sure to note which are the key ideas and group the chunks/details around those. For example, the proof of Lagrange's Theorem requires the conceptual chunks of
- consider all (left) cosets of H
- show they partition G (they cover G and are pairwise disjoint)
- note they all have the same size

and the sticky details are mainly notational.

4. For each conceptual chunk and sticky detail, write a question whose answer is likely to give a student insight as to how to proceed toward the curricular goal. Again, focus on key ideas so that students can see them as highlights, rather than viewing the mathematical narrative as flat. For example, if students have already conjectured Lagrange's Theorem, such questions could be

(a) For $g \in G$ and $H < G$, what is $|gH|$?
(b) Compare $|g_1H|$ to $|g_2H|$.
(c) What kind of subset of G is $g_1H \cap g_2H$? Is it a subgroup?
(d) What is $|g_1H \cap g_2H|$?
(e) [I think there should be another one here but I'm not sure what it is]
(f) Prove that if $H < G$ are finite, then $|H|$ divides $|G|$.

5. Add questions or instructions that generate examples or request conjectures or both.

6. Read over your question list and revise it to address any gaps or any places where there are too many successive small questions.

7. Give your worksheet a title.

Guide 2: How to convert a worksheet into a MathILy-style class lesson

The purpose of creating a worksheet is to make clear in your own mind what ingredients are needed for students to learn a concept, and how learning that concept might be broken down into smaller chunks that are more accessible to students. Once this has been achieved, you have a mathematical framework set as a starting point.

However, there is a presentational framework that needs to be prepared as well. Is there some way to set the original question into a silly story (if introducing a topic)? How much technical terminology is needed to advance the concepts, and how do you want students to develop it—or would it be better for you to feed them the terminology?

Try to anticipate what students may do with the tasks you set, so that you will have some reactions prepared. After a few days of class, you should also think about: For your particular class, are students likely to make conjectures quickly or need to be prompted? Will some students jump into proofs while others sit back?

The answers to all of these questions will inform your classroom plan. Consider what material you want to verbally present and write on the board to set up an initial question or situation, and how you will elicit questions/conjectures from the students. You may hope that they will volunteer ideas immediately, or expect that you will break them into groups to brainstorm. Are there exercises that you can ask them to complete, or examples they can generate, or previously generated data that they can examine?

Think through what interplay you would prefer: Do you want to mainly be at the board, or mainly circulating among groups (who then present ideas), or do you want to alternate, or...?

Be prepared to interrogate student presentation and thinking—at the board and in small groups. Likewise, think about what will constitute enough frustration/wheel-spinning that you want to manually break down a problem further for the students.

Guide 3: How to plan a few hours of MathILy-style class lessons
(a.k.a. how to plan a short IBL course with appropriate curriculum and narrative arc)

Presumably you are already familiar with how to construct a MathILy class lesson from Guide 2. There you worked with a single short topic that is one chunk in a larger thread. But you may want to address a longer/larger topic over a few class periods, or teach a short extracurricular course (for example to a Math Circle or Math Club). For the purposes of this guide, we will suppose you have 5 class sessions to devote to this topic. What to do?

First, brainstorm one or more topics. These are likely to fall under one of the two following broad categories: (a) a selection of disjoint but related topics/questions, and (b) a sequence of activities building toward a result. Experienced instructors suggest that (a) is easier to execute when one is less comfortable with the material, and more robust if things go awry.

Here are two examples.
For type (a) above, the topic "combinatorial optimization" might be broken into chunks by problem and solution types (e.g. assignment problem, knapsack problem, facility location problem, simplex, branch-and-bound) with ideas and definitions introduced incrementally alongside (process of optimization, integer/linear programs, abstract combinatorial optimization problem).
For type (b) above, the topic "classification of surfaces" might be broken into chunks by cognitive steps (definitions and examples, surgery experiments, specific reductions and typing, general reductions and typing, conjecture and proof of classification theorem).

Back to your topic choice: Ask yourself the following questions about a topic choice; they will help you know whether or not it is appropriate and also assist you in preparing classes or a short course.

What is the essential lesson, or what are the essential ideas, that you want students to take away from this class thread? Is there (in some sense) a punch line? What material is prerequisite to this topic?

Try to break your topic down into chunks (of one class meeting each). You are allowed exactly 5 chunks. Then try to plan a class for each of the five chunks, using your usual MathILy lesson planning protocols: work backwards from the class goals to motivating questions and activities that are accomplishable within the given timeframe. Keep in mind that out-of-class work can become unbounded (or not done!), so anything you want them to work through should be done during one of the 5 chunks. Also keep in mind that students may not have mastered the prerequisite material, so you may need to build in some redevelopment or further development of that material.

Now for a new round of questions: Were you overambitious? (The answer to this is probably 'yes.') If so, think about whether this is a fatal overambitiousness or a mitigatable overambitiousness. Are there subtopics that can be skipped? Or can the overall narrative be shortened to a less ambitious goal? Will the entire point of the topic/course be deflated if content is reduced?

Once you have a satisfactory curriculum and narrative arc, think about how you can alter this on the fly depending on what happens in class. If the students are a bit faster or slower than you expect, are there other natural ways to re-chunk the material?